Questions 1, 2 and 3 each weigh 1/3. These weights, however, are only indicative for the overall evaluation.

Henrik Jensen
Department of Economics
University of Copenhagen

MONETARY POLICY
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QUESTION 1:

Evaluate whether the following statements are true or false. Explain your answers.

(i) In the Lucas Island model with imperfect information, higher "local" monetary volatility, relative to "aggregate" volatility, makes the real effect of aggregate money shocks smaller.

A **False.** When there are relatively more "local" volatility, it becomes more likely that an observed monetary shock is local. Hence, agents on every island will respond stronger to an aggregate monetary shock (e.g., by increasing work more).

(ii) By the logic of the Poole (1970) model, the United States' Federal Reserve normally uses the nominal interest rate as the policy instrument because the volatility of money-market shocks are negligible.

A **False.** The Poole model shows that in the choice between using the money supply or the nominal interest rate as the operating procedure, the relative variability on goods and money markets plays a central role. An interest-rate operating procedure insulates output from money-market shocks whereas a money-supply operating procedure reduces the impact of goods demand shocks. Adopting an interest-rate operating procedure is thus advantageous when money-market shocks are relatively more dominant than goods market shocks. Usage of the nominal interest as a policy instrument would therefore be a poor choice according to the Poole model if there is basically no volatility of money-market shocks.

(iii) The optimal inflation rate in the simple New-Keynesian model witch sticky goods prices is given by the Friedman rule.

A **False.** In the simple New-Keynesian model with sticky goods prices, inflation is costly as it causes an inefficient dispersion of the demand and production of monopolistically competitive goods. This happens as only a fraction of firms changes prices in any period. This inefficient dispersion is only eliminated if no prices change, i.e., when inflation is zero. Under the Friedman rule, prices are falling at a rate equal to the real interest rate; this will also cause undesirable demand dispersion, and is therefore inoptimal.

QUESTION 2:

Consider an infinite-horizon economy in discrete time, where utility of the representative agent is given by

$$U = \sum_{t=0}^{\infty} \beta^t u(c_t, n_t), \qquad 0 < \beta < 1, \tag{1}$$

with

$$u(c_t, n_t) \equiv \log c_t - \frac{1}{1+\eta} n_t^{1+\eta}, \qquad \eta > 0,$$

where c_t is consumption and n_t is employment. Agents face the budget constraint

$$c_t + m_t + b_t = f(n_t) + \tau_t + \frac{1+i_{t-1}}{1+\pi_t} b_{t-1} + \frac{1}{1+\pi_t} m_{t-1}, \tag{2}$$

where m_t is real money balances at the end of period t, b_t is real bond holdings, τ_t denotes real monetary transfers from the government, i_t is the nominal interest rate on bonds, and π_t is the inflation rate. Function f is defined as

$$f(n_t) \equiv A n_t^{1-\alpha}, \qquad A > 0, \quad 1 > \alpha > 0.$$

Agents also face a cash-in-advance constraint

$$c_t \leq \frac{1}{1+\pi_t} m_{t-1} + \tau_t. \tag{3}$$

(i) Derive the relevant conditions for optimal behavior of the representative agent. For this purpose, set up the value function

$$V\left(m_{t-1},b_{t-1}\right)=\max_{c_t,n_t,m_t}\left\{u\left(c_t,n_t\right)+\beta V\left(m_t,b_t\right)-\mu_t\left(c_t-\frac{1}{1+\pi_t}m_{t-1}-\tau_t\right)\right\},$$

where b_t is eliminated by use of (2), and where μ_t is the Lagrange multiplier on (3). Interpret intuitively the first-order conditions for c_t, n_t, m_t and the expressions for the partial derivatives of V.

A The first-order conditions are given by

$$u_c\left(c_t,n_t\right)-\mu_t-\beta V_b\left(m_t,b_t\right) = 0, \tag{*}$$

$$u_n\left(c_t,n_t\right)+\beta V_b\left(m_t,b_t\right)\left(1-\alpha\right)An_t^{-\alpha} = 0, \tag{**}$$

$$\beta V_m\left(m_t,b_t\right)-\beta V_b\left(m_t,b_t\right) = 0. \tag{***}$$

The first-order condition for consumption, (*), shows that the marginal utility of consumption is equated to the marginal costs which are the liquidity cost arising from the CIA constraint (as consumption requires money) and the marginal value of future lost savings. (**) shows that employment are decided so as to equate the marginal utility loss with the marginal value of future savings times the income generated from employment (which is the marginal product of labor). Finally, money is held so as to equate its future marginal value with the marginal value of future lost savings; cf. (***).

The partial derivatives of the value function, using the Envelope Theorem, is found as:

$$V_m\left(m_{t-1},b_{t-1}\right) = \mu_t\frac{1}{1+\pi_t}+\beta V_b\left(m_t,b_t\right)\frac{1}{1+\pi_t}, \tag{****}$$

$$V_b\left(m_{t-1},b_{t-1}\right) = \beta V_b\left(m_t,b_t\right)\frac{1+i_{t-1}}{1+\pi_t}. \tag{*****}$$

As seen by (****), the marginal value of money held at the end of $t-1$, is determined by the next-period liquidity value (μ_t) and the marginal value of future savings (both corrected for inflation). Finally, (*****) shows that the marginal value of savings held at the end of $t-1$, is equal to the next-period marginal value times the real interest rate—a version of the conventional Keynes-Ramsey rule.

(ii) Show that

$$i_t=\frac{\mu_{t+1}}{\beta V_b\left(m_{t+1},b_{t+1}\right)}, \tag{4}$$

and

$$-\frac{u_c(c_t,n_t)}{u_n(c_t,n_t)} = \frac{(\mu_t/[\beta V_b(m_t,b_t)])+1}{(1-\alpha)An_t^{-\alpha}}. \tag{5}$$

Discuss (4), and explain how nominal interest rate changes affect the labor supply decision (5).

A Combining (***) and (****) we get

$$V_b(m_t,b_t) = \frac{\mu_{t+1}+\beta V_b(m_{t+1},b_{t+1})}{1+\pi_{t+1}}.$$

Forwarding (*****) one period yields

$$V_b(m_t,b_t) = \beta V_b(m_{t+1},b_{t+1})\frac{1+i_t}{1+\pi_{t+1}}.$$

Equating the right-hand sides gives

$$\begin{aligned}\beta V_b(m_{t+1},b_{t+1})\frac{1+i_t}{1+\pi_{t+1}} &= \frac{\mu_{t+1}+\beta V_b(m_{t+1},b_{t+1})}{1+\pi_{t+1}},\\ \beta V_b(m_{t+1},b_{t+1})(1+i_t) &= \mu_{t+1}+\beta V_b(m_{t+1},b_{t+1}),\end{aligned}$$

and thus

$$i_t = \frac{\mu_{t+1}}{\beta V_b(m_{t+1},b_{t+1})},$$

which is (4). From (*) and (**), we get

$$-\frac{u_c(c_t,n_t)}{u_n(c_t,n_t)} = \frac{\mu_t+\beta V_b(m_t,b_t)}{\beta V_b(m_t,b_t)(1-\alpha)An_t^{-\alpha}},$$

and thus

$$-\frac{u_c(c_t,n_t)}{u_n(c_t,n_t)} = \frac{(\mu_t/[\beta V_b(m_t,b_t)])+1}{(1-\alpha)An_t^{-\alpha}},$$

which is (5).

Equation (4) shows that the cash-in-advance constraint is binding when the nominal interest rate is positive. This follows as the nominal interest rate is the opportunity cost of holding money. Hence, a positive interest rate gives consumers the incentive to only hold the necessary cash, and as a result the constraint is binding. Equation (5) shows the marginal rate of substitution between consumption and employment, and rewritten by use of (4), one gets

$$-\frac{u_c(c_t,n_t)}{u_n(c_t,n_t)} = \frac{i_{t-1}+1}{(1-\alpha)An_t^{-\alpha}}.$$

A higher nominal interest rate makes consumption less desirable relative to leisure. One can interpret the nominal interest rate as a "tax" on consumption (that has to be paid in form of being forced to hold non-interest rate bearing money in order to consume); one that leisure is not subject to. Therefore, a higher nominal interest rate will cause a substitution away from consumption.

(iii) Consider the steady state. Apply the particular functional form of u and use (4)–(5) to derive the solution for employment as a function of the nominal interest rate, using that the national account is $c_t = An_t^{1-\alpha}$. What is the optimal steady-state value of the nominal interest rate? [Hint: Use that optimal steady-state employment, n, solves $\max_n \left\{\log\left(An^{1-\alpha}\right) - \frac{1}{1+\eta}n^{1+\eta}\right\}$.] Explain.

A We have from (4) and (5) that in steady state

$$\begin{aligned} -\frac{u_c(c,n)}{u_n(c,n)} &= \frac{(\mu/[\beta V_b(m,b)])+1}{(1-\alpha)An^{-\alpha}} \\ &= \frac{1+i}{(1-\alpha)An^{-\alpha}}. \end{aligned}$$

With the specific functional form of utility:

$$\frac{1}{c}\frac{1}{n^{\eta}} = \frac{1+i}{(1-\alpha)An^{-\alpha}}$$

Now use that the national account is $c_t = y_t = An_t^{1-\alpha}$ to get

$$\begin{aligned} \frac{1}{cn^{\eta}} &= \frac{1+i}{(1-\alpha)(c/n)}, \\ n^{-(1+\eta)} &= \frac{1+i}{1-\alpha}, \\ n &= \left(\frac{1+i}{1-\alpha}\right)^{-\frac{1}{1+\eta}} \end{aligned} \qquad (\Delta)$$

We see here that employment (and thus consumption) is negatively related to the nominal interest rate. The explanation is as above: The interest rate makes consumption relatively more expensive, leading to a decline in labor supply and employment.

The optimal steady-state employment level is found as the one that maximizes utility:

$$\max_n \quad \log\left(\underbrace{An^{1-\alpha}}_{=c}\right) - \frac{1}{1+\eta}n^{1+\eta}.$$

The first-order condition is given by

$$\frac{1-\alpha}{n} = n^{\eta},$$

and thus

$$n = (1-\alpha)^{\frac{1}{1+\eta}}.$$

Comparing this with (Δ), it follows that the optimal nominal interest rate is zero since this will replicate the optimum. I.e., the Friedman rule is desirable in this economy, as it eliminates the distortion of the consumption-leisure decision.

QUESTION 3:

Consider the following New-Keynesian log-linear model of a closed economy:

$$\begin{aligned} y_t &= \mathrm{E}_t y_{t+1} - \sigma^{-1}\left(\widehat{i}_t - \mathrm{E}_t \pi_{t+1}\right), \qquad \sigma > 0, &(1)\\ \pi_t &= \beta \mathrm{E}_t \pi_{t+1} + \kappa\left(y_t - y_t^n\right) \qquad 0 < \beta < 1, \quad \kappa > 0, &(2)\\ \widehat{i}_t &= \phi \pi_t, \qquad \phi > 1, &(3)\end{aligned}$$

where y_t is output, $\widehat{i}_t$ is the nominal interest rate's deviation from steady state, and π_t is goods-price inflation, y_t^n is the natural rate of output, which is assumed to be a mean-zero i.i.d. shock. E_t is the rational-expectations operator conditional upon all information up to and including period t.

(i) Derive the solutions for y_t, π_t and $\widehat{i}_t$. [Hint: Conjecture that the solutions are linear functions of y_t^n, and use the method of undetermined coefficients.] Explain how the shock is transmitted onto the variables.

A Conjecture the following solutions:

$$\begin{aligned} y_t &= A y_t^n,\\ \pi_t &= B y_t^n.\end{aligned}$$

Forward these and take expectations:

$$\begin{aligned} \mathrm{E}_t y_{t+1} &= A \mathrm{E}_t y_{t+1}^n = 0,\\ \mathrm{E}_t \pi_{t+1} &= B \mathrm{E}_t y_{t+1}^n = 0.\end{aligned}$$

Insert the conjectures, and expected one-period ahead conjectures [and (3)] into (1) og (2) and get:

$$\begin{aligned} y_t &= -\sigma^{-1}\phi\pi_t, \\ \pi_t &= \kappa\left(y_t - y_t^n\right), \end{aligned}$$

and thus

$$\begin{aligned} Ay_t^n &= -\sigma^{-1}\phi By_t^n, \\ By_t^n &= \kappa\left(Ay_t^n - y_t^n\right). \end{aligned}$$

The unknown coefficients are therefore determined by

$$\begin{aligned} A &= -\sigma^{-1}\phi B, \\ B &= \kappa\left(A - 1\right), \end{aligned}$$

leading to

$$\begin{aligned} A &= -\sigma^{-1}\phi\kappa\left(A - 1\right), \\ A\left(1 + \sigma^{-1}\phi\kappa\right) &= \sigma^{-1}\phi\kappa \\ A &= \frac{\sigma^{-1}\phi\kappa}{1 + \sigma^{-1}\phi\kappa} \\ B &= -\frac{\kappa}{1 + \sigma^{-1}\phi\kappa} \end{aligned}$$

Hence, the solutions for y_t, π_t and $\widehat{i}_t$ are given by

$$\begin{aligned} y_t &= \frac{\sigma^{-1}\phi\kappa}{1 + \sigma^{-1}\phi\kappa} y_t^n, \\ \pi_t &= -\frac{\kappa}{1 + \sigma^{-1}\phi\kappa} y_t^n, \\ \widehat{i}_t &= -\frac{\phi\kappa}{1 + \sigma^{-1}\phi\kappa} y_t^n. \end{aligned}$$

A positive shock to the natural rate of output, reduces the marginal cost of producers leading to lower inflation. By the nominal interest-rate rule, the nominal and real interest rate is reduced, which increases output.

(ii) Assume that stabilizing the output gap, $y_t - y_t^n$, and π_t is preferable. Discuss the underlying model's welfare rationale for this assumption.

A In this model, the output gap is proportional to the real marginal costs of producers. Nominal rigidities causes fluctuations in real marginal costs and thus employment which are undesirable. A stable output gap is synonymous with stable real marginal costs, which eliminates this distortion of nominal rigidities. Moreover, nominal rigidities imply that any inflation different from zero causes relative price changes and thus inefficient demand dispersion of the various consumer goods in the economy.

(iii) Evaluate formally whether stabilizing $y_t - y_t^n$ and π_t at the same time is possible in the model by appropriate choice of ϕ. Discuss.

A Since this model only has shocks to the natural rate, the "divine coincidence" applies. I.e., it is possible to stabilize both the output gap and inflation. To see this, write the solutions of the output gap and inflation

$$\begin{aligned} y_t - y_t^n &= \frac{\sigma^{-1}\phi\kappa}{1+\sigma^{-1}\phi\kappa} y_t^n \\ &= -\frac{1}{1+\sigma^{-1}\phi\kappa} y_t^n, \\ \pi_t &= -\frac{\kappa}{1+\sigma^{-1}\phi\kappa} y_t^n. \end{aligned}$$

Letting the central bank respond vigorously towards inflation, $\phi \to \infty$, will accomplish full stabilization of both the output gap and inflation; the denominator $1+\sigma^{-1}\phi\kappa$ in both expressions goes to infinity, so $\lim_{\phi\to\infty} y_t - y_t^n = \lim_{\phi\to\infty} \pi_t = 0$. In equilibrium, the nominal interest rate will be

$$\widehat{i}_t = -\sigma y_t^n,$$

which can be interpreted as a policy following the natural rate of interest (the real interest rate compatible with output at the natural rate).